B. Application to Uniaxial Strain in Quartz

We assume the deformation to occur in the X direction only. The coordinate transformation is accordingly,

$$x_1 = (1 - \gamma)a_1$$

$$x_2 = a_2$$

$$x_3 = a_3$$

Formulas (2.14) through (2.18) then give:

$$J = V/V_0 = 1 - \gamma.$$

$$N_1 = \gamma(\gamma/2 - 1)$$

$$\rho_0[E - E_0] = 1/2\ c_{11}\ N_1^{\ 2} + 1/6\ c_{111}\ N_1^{\ 3} + 1/24\ c_{1111}\ N_1^{\ 4} + \ldots$$

$$t_k = c_{1k}\ N_1 + 1/2\ c_{11k}\ N_1^{\ 2} + 1/6\ c_{111k}\ N_1^{\ 3} + \ldots\ (k = 1, 2 \ldots 6)$$

or writing out the components:

$$t_1 = c_{11}\ N_1 + 1/2\ c_{111}\ N_1^{\ 2} + 1/6\ c_{1111}\ N_1^{\ 3} +$$

$$t_2 = c_{12}\ N_1 + 1/2\ c_{112}\ N_1^{\ 2} + \ldots$$

$$t_3 = c_{13}\ N_1 + 1/2\ c_{113}\ N_1^{\ 2} + \ldots$$

$$t_4 = c_{14}\ N_1 + 1/2\ c_{114}\ N_1^{\ 2} + \ldots$$

$$t_5 = c_{15}\ N_1 + 1/2\ c_{115}\ N_1^{\ 2} + \ldots$$

$$t_6 = c_{16}\ N_1 + 1/2\ c_{116}\ N_1^{\ 2} + \ldots$$

The stress components are then:

$$\sigma_1 = (1-\gamma)t_1 \qquad \sigma_4 = (1-\gamma)^{-1}\, t_4$$
$$\sigma_2 = (1-\gamma)^{-1}\, t_2 \qquad \sigma_5 = t_5 \qquad (2.20)$$
$$\sigma_3 = (1-\gamma)^{-1}\, t_3 \qquad \sigma_6 = t_6$$

For alpha quartz compressed in the X-direction the above formulas are correct as they stand. For compression in other directions the proper translation of subscripts must, of course, be made to indicate the correct constants.

The above formulas have been applied to uniaxial compression of X and Z-cut quartz, using the second and third order constants determined by McSKIMIN, _et al._ (39) and THURSTON, _et al._ (40). Values of these constants are shown in Table III.

The resulting curves are plotted in Figs. 2.6, 2.7 and 2.9. The values of shock velocity, $U_s$ and particle velocity, $u_p$, of Fig. 2.6 were obtained from the Hugoniot relations:

$$U_s = V_0\left(\frac{\sigma}{V_0 - V}\right)^{1/2}$$

and

$$u_p = [\sigma(V_0 - V)]^{1/2} \quad .$$

The predictions are seen to fall outside the estimated error of the shock data, indicating that the fourth-order term contributes significantly to the energy (and the stress) at larger values of strain. Although it might be thought that the discrepancy is due in part to the use of isentropic second-order moduli and mixed isentropic-isothermal third-order moduli to predict Hugoniot states, for which internal energy is greater than for isentropic compression, a straightforward calculation shows that the errors